The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam ***Solutions 2.0***

Date: December 10, 2021 Course: EE 313 Evans

Name: **Things,** **Stranger**
Last, First

- This in-person exam is scheduled to last three hours.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Prob	Point Value	Average score	Standard Deviation	Topic	
1	16	**14.38**	**1.83**	Heart and Soul of Discrete-Time Signals & Systems	**Jim Hopper**
2	12	**8.75**	**2.81**	Continuous-Time Fourier Series	**Dustin**
3	12	**9.56**	**1.64**	Discrete-Time Audio Signal Processing	**Max**
4	12	**10.56**	**1.29**	Continuous-Time Communication System	**Mike Wheeler**
5	12	**7.91**	**2.02**	Discrete-Time Filter Analysis	**Joyce Byers**
6	12	**10.03**	**1.98**	Continuous-Time Signal Acrobatics	**Will**
7	12	**9.56**	**2.37**	Discrete-Time Equalization	**Lucas**
8	12	**7.75**	**3.82**	Continuous-Time Frequency-Domain Analysis	**Eleven**
Total	100	**78.50**	**11.41**		

Note: 32 students took the final exam. The median score was 81. A 4-point offset was added to each student's score which brought the median score to 85 and the average score to 82.50.

Problem 1. Heart and Soul for Discrete-Time Signals and Linear Systems. *16 points.*

(a) **LTI Systems**. Consider a discrete-time linear time-invariant system with input signal $x[n]$, impulse response $h[n]$ and output signal $y[n]$. 9 *points.*

i. Give the relationship for $y[n]$ to $x[n]$ and $h[n]$ involving only operations in the discrete-time domain.

ii. Give the relationship for $Y_{freq}(\hat{\omega})$ to $X_{freq}(\hat{\omega})$ and $H_{freq}(\hat{\omega})$ using only operations in the discrete-time Fourier (frequency) domain.

iii. Give the relationship for $Y(z)$ to $X(z)$ and $H(z)$ using only operations in the z-domain.

	Input	System	Output	
Discrete-time domain	$x[n]$ →	$h[n]$	→ $y[n]$	$y[n] =$
Discrete-time Fourier domain	$X_{freq}(\hat{\omega})$	$H_{freq}(\hat{\omega})$	$Y_{freq}(\hat{\omega})$	$Y_{freq}(\hat{\omega}) =$
Z-domain	$X(z)$	$H(z)$	$Y(z)$	$Y(z) =$

Fall 2018 Final Exam Prob 1
Lecture slide 8-8 time domain
Lecture slides 9-4 to 9-11 freq.
Lecture slide 10-7 z-domain
SPFirst Sec. 5-6.1 pp. 118-210
SPFirst Sec. 6.1 pp. 130-132
SPFirst Sec. 7.5 p. 171 eq. (7.19)

$\boldsymbol{y[n] = h[n] * x[n]}$ **-OR-** $\boldsymbol{y[n] = x[n] * h[n]}$

$\boldsymbol{Y_{freq}(\hat{\omega}) = H_{freq}(\hat{\omega})\, X_{freq}(\hat{\omega})}$ **-OR-** $\boldsymbol{Y_{freq}(\hat{\omega}) = X_{freq}(\hat{\omega})\, H_{freq}(\hat{\omega})}$

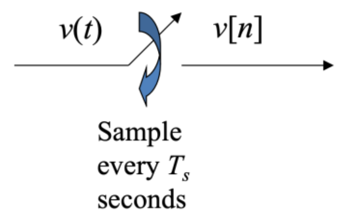
$\boldsymbol{Y(z) = H(z)\, X(z)}$ **-OR-** $\boldsymbol{Y(z) = X(z)\, H(z)}$ **where** $\boldsymbol{\text{ROC}\{Y(z)\} = \text{ROC}\{X(z)\} \cap \text{ROC}\{H(z)\}}$

(b) **Sampling in the time domain.** *7 points.*

Consider the continuous-time sinusoidal signal $v(t)$ at fixed frequency f_0 in Hz defined as

$$v(t) = \cos(2\pi f_0 t)$$

observed for $-\infty < t < \infty$ and sampled at sampling rate f_s to produce signal $v[n]$ as shown on the right.



i. Give a formula for $v[n]$ observed for $-\infty < n < \infty$. *3 points.*

$$\boldsymbol{v[n] = v(t)|_{t=nT_s} = \cos\left(2\pi f_0 (nT_s)\right)}$$

$$\boldsymbol{v[n] = \cos(2\pi f_0 T_s n) = \cos\left(2\pi \frac{f_0}{f_s} n\right) = \cos(\hat{\omega}_0 n)}$$

Fall 2018 & 2021 Midterm 1.1(a)
SPFirst Sec. 4-1.1 & lec. slide 5-5
Homework 3.2 & 4.1

ii. Give a formula for the discrete-time frequency $\hat{\omega}_0$ for $v[n]$ in rad/sample in terms of the continuous-time frequency f_0. *2 points.*

$$\boldsymbol{\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}}$$

Fall 2018 & 2021 Midterm 1.1(b)
SPFirst Sec. 4-1.1 & lec. slide 5-5
Homework 3.2 & 4.1

iii. What continuous-time frequencies f are captured by sampling at sampling rate f_s without aliasing? *2 points.*

Sampling Theorem says "Continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed from its samples $x(n\,T_s)$ if samples taken at rate $f_s > 2\,f_{max}$" (Lecture Slide 16-6).

So, $f_{max} <$ (½) f_s which means $-\frac{1}{2} f_s < f < \frac{1}{2} f_s$ when including negative frequencies.

Fall 2017 Final Exam Prob 1 | Fall 2021 Midterm 1.2 & 1.3
Fall 2018 Midterm 1.3 | SPFirst Sec. 11-4.6 Ex. 11-4 p. 321
SPFirst Sec. 3-3 to 3-6
Lecture Slides 3-7 to 3-14
In-Lecture Assignment #4
Mini-Project #1

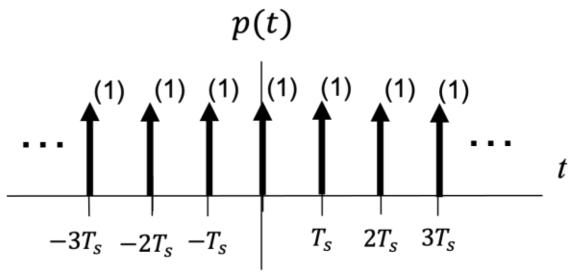
Problem 2. Continuous-Time Fourier Series. *12 points.*

A continuous-time impulse train can model the periodic instantaneous closing and opening of a switch in sampling when viewing the sampling output in continuous time.

For a sampling time of T_s, the impulse train can be expressed as

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s)$$

and its plot is



where (1) indicates that the area under each Dirac delta is 1.

(a) What is the fundamental period T_0 of $p(t)$? *2 points.*

Lecture Slides 2-3 & 3-4
SPFirst Sec. 3-3

A Dirac delta occurs every T_s seconds.

Fundamental period is $T_0 = T_s$

(b) Compute the Fourier series coefficients using the Fourier synthesis formula. *6 points.*

$$p(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j\,2\,\pi\,(k\,f_0)\,t}$$

where

$$a_0 = \frac{1}{T_0} \int_{-\frac{1}{2}T_0}^{\frac{1}{2}T_0} p(t)dt$$

$$a_k = \frac{1}{T_0} \int_{-\frac{1}{2}T_0}^{\frac{1}{2}T_0} p(t)\, e^{-j\,2\,\pi\,(k\,f_0)\,t}\, dt$$

The limits of integration are from $-\frac{1}{2}T_0$ to $\frac{1}{2}T_0$ to make sure to include the Dirac delta at the origin inside the limits.

In the fundamental period $-\frac{1}{2}T_s < t \le \frac{1}{2}T_s$, there is one Dirac delta at $t = 0$. Fourier series coefficient a_0 is the average value of the signal over the fundamental period:

$$a_0 = \frac{1}{T_s} \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} p(t)\, dt = \frac{1}{T_s} \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} \delta(t)\, dt = \frac{1}{T_s}$$

We're using the fact that the Dirac delta has unit area.
We can use the sifting property for the Dirac delta for a_k:

$$a_k = \frac{1}{T_s} \int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} \delta(t)\, e^{-j\,2\,\pi\,(k\,f_0)\,t}\, dt = \frac{1}{T_s}$$

The sifting property is

$$\int_{-\infty}^{\infty} g(t)\, \delta(t)\, dt = g(0)$$

(c) Plot the spectrum of the Fourier series. *2 points.*

All Fourier series coefficients have the same value.

(d) Describe the spectrum of the Fourier series coefficients. *2 points.*

The spectrum is constant for all harmonic frequencies kf_s. Looks like an impulse train.

Lecture slides 5-5 to 5-6 and 9-12 to 9-13

SPFirst Sec. 4-1.1

Problem 3. Discrete-Time Audio Signal Processing. *12 points.*

(a) Consider generating an A major chord by playing the notes A, C# and E at the same time where the note frequencies are f_A = 440 Hz, $f_{C\#}$ = 550 Hz and f_E = 660 Hz, respectively:

$$x(t) = \cos(2\,\pi\, f_A\, t) + \cos(2\,\pi\, f_{C\#}\, t) + \cos(2\,\pi\, f_E\, t)$$

1. Determine the corresponding discrete-time frequencies $\hat{\omega}_A$, $\hat{\omega}_{C\#}$ and $\hat{\omega}_E$ for a sampling rate of f_s = 44100 Hz. *3 points*. **Discrete-time frequencies are in units of rad/sample.**

$$\hat{\omega}_A = 2\pi \frac{f_A}{f_s} = 2\pi \frac{440 \text{ Hz}}{44100 \text{ Hz}} = 2\pi \frac{22}{2205}$$

$$\hat{\omega}_{C\#} = 2\pi \frac{f_A}{f_s} = 2\pi \frac{550 \text{ Hz}}{44100 \text{ Hz}} = 2\pi \frac{11}{882}$$

$$\hat{\omega}_E = 2\pi \frac{f_A}{f_s} = 2\pi \frac{660 \text{ Hz}}{44100 \text{ Hz}} = 2\pi \frac{11}{735}$$

Handout D Discrete-Time Periodicity

Fall 2021 Midterm 1.1

2. What is the smallest discrete-time period in samples for $x[n]$? *3 points*.

For a discrete-time cosine signal with discrete-time frequency in the form of $\omega_0 = 2\pi \frac{N}{L}$ where N and L are relatively prime integers, the smallest discrete-time period is L samples.

Discrete-time periods for the above discrete-time frequencies are 2205, 882 and 735 samples.

The smallest discrete-time period for $x[n]$ is the lcm(2205, 882, 735) = 4410 samples.

(b) A discrete-time signal with sampling rate of f_s of 8000 Hz has the following "UX" spectrogram. The spectrogram was computed using 1000 samples per block and an overlap of 900 samples.

SPFirst Sec. 3-7 & 3-8

Lecture 4 Slides

Homework 2.3

Mini-Project #1

In-Lecture Assignments 1, 4 & 5

1. Describe the frequency content vs. time. *3 points*.

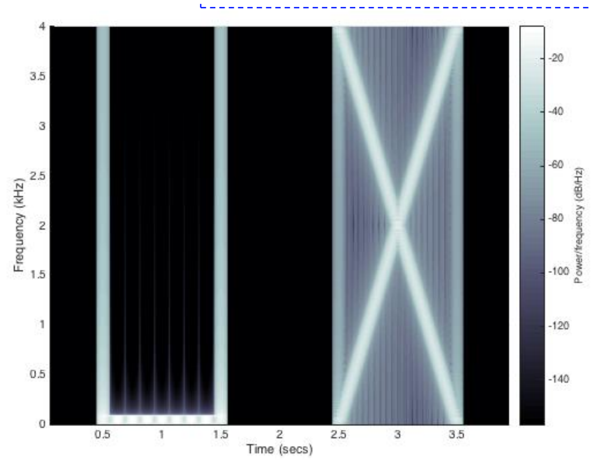
By using the intensity scale shown to the right of the spectrogram plot:

***t* = 0.5s : all frequencies present**

0.5s < *t* < 1.5s : Low frequencies 0 to 0.1 kHz continuously present (in white) plus six less intense short bursts of frequencies 0 to 1 kHz equally spaced in time (short rect. pulses)

***t* = 1.5s : all frequencies present**

2.5s < *t* < 3.5s : chirp increasing from 0 to ½f_s plus a chirp decreasing from ½f_s to 0



2. What would the signal sound like when played as audio signal? *3 points*.

0.5s < *t* < 1.5s : Bass tones 20-100 Hz plus lower intensity 0-1 kHz freq. repeated 6 times.
Clicking sounds at 0.5s and 1.5s

2.5s < *t* < 3.5s : Note increasing 0 to 4 kHz, and note decreasing 4 to 0 kHz, with time.

Problem 3(b) Supplemental information not expected for students to have provided in their answers

Matlab code to generate the spectrogram.

```
fs = 8000;
Ts = 1 / fs;
tmax = 4;
utSignal = zeros(1, tmax*fs);
t1sec = 0 : Ts : (1 - Ts);
%% Spectrogram parameters
Nfft = 1000;
Noverlap = 900;
%% Generate low frequency groups
f0 = fs / Nfft;
lowfcosines = zeros(1, length(t1sec));
for n = 1 : 10
  f1 = n*f0;
  lowfcosines = lowfcosines + cos(2*pi*f1*t1sec);
end
%% Create chirp signals
fstart = 0;
fend = fs/2;
fstep = fend - fstart;
phi = pi*fstep*(t1sec.^2);
upchirp = cos(2*pi*fstart*t1sec + phi);
downchirp = cos(2*pi*fend*t1sec - phi);
%% Draw U into spectrogram
utSignal(0.5*fs+1:1.5*fs) = lowfcosines;
%% Draw X into spectrogram
utSignal(2.5*fs+1:3.5*fs)= upchirp + downchirp;
%% Plot the spectrogram
spectrogram(utSignal, hamming(Nfft), Noverlap, Nfft, fs, 'yaxis');
colormap bone;
```

Matlab code to play the signal as an audio signal. Playing the sound over laptop speakers won't likely play sub-woofer frequencies (20-200 Hz) which is the bulk of the signal from 0.5s to 1.5s. You might try to use headphones, or find a system with a sub-woofer.

```
soundsc(utSignal, fs);
```

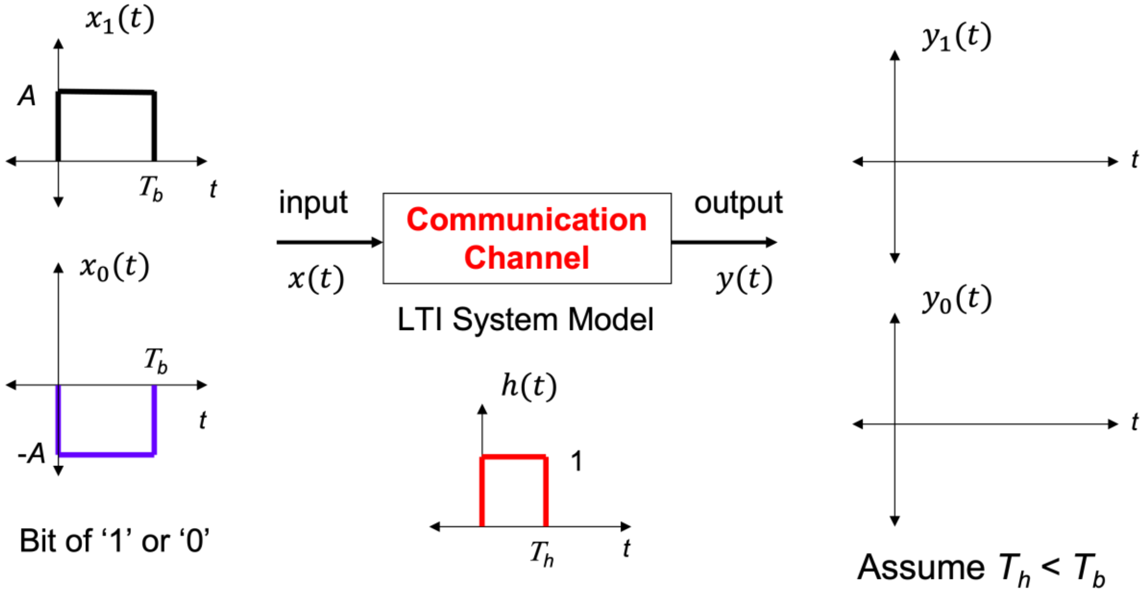
Summer 2016 Final Exam Prob. 7(b) | Fall 2017 Midterm 2.2(a) | Fall 2017 Final Exam Prob. 3 | Fall 2018 Midterm 2.1(a)
SPFirst Sec. 9-7 Convolution | Lecture Slide 12-6 Rect Pulse | Lecture 13 CT Convolution | Homework 7.4
Handout E Convolving Rect Pulses

Problem 4. Continuous-Time Communication System. *12 points.*

We will transmit one bit over a communication channel and analyze the result at the receiver.

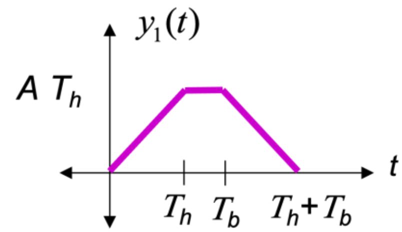
- A bit of value '1' will be transmitted as $x_1(t)$, a rectangular pulse of positive amplitude A.
- A bit of value '0' will be transmitted as $x_0(t)$, a rectangular pulse of negative amplitude -A.

We will model the communication channel as an LTI system, as given below.



(a) Plot $y_1(t) = h(t) * x_1(t)$. Label the important points on the vertical and horizontal axes in terms of A, T_b, and T_h. Assume $T_h < T_b$. *4 points.*

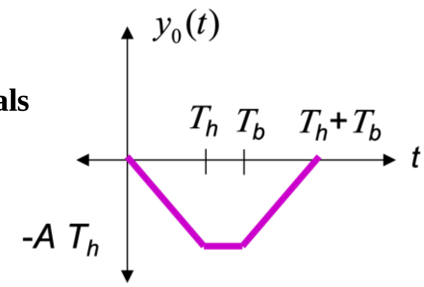
In-Lecture Assignment 8



(b) Plot $y_0(t) = h(t) * x_0(t)$. Label the important points on the vertical and horizontal axes in terms of A, T_b, and T_h. Assume $T_h < T_b$. *4 points.*

We can use the convolution property in which scaling one of the signals being convolved causes the same scaling in the convolution result.

That is, we can reuse the result from (a) by multiplying $x_1(t)$ by -1.



(c) Determine how the receiver could reliably determine which bit had been transmitted by processing $y(t)$. *4 points.*

Mini-Project #2

Approach #1: **Sample the received signal $y(t)$ at $t = T_b$ and compare the amplitude value against 0 to decide what bit was most likely to have been sent.**

Approach #2: **Apply a matched filter (from problem 6 below) to $y(t)$ and sample the result at T_b seconds and compare the amplitude value against 0.**

Approach #3: **Apply an equalizer to compensate for the distortion in the communication channel modeled by $h(t)$ and then apply approach #1 or #2.**

Summer 2016 Final Exam Probs 4 & 8 | Fall 2017 Midterm 2.3 & 2.4
Fall 2018 Midterm 2.3 | Fall 2018 Final Exam Prob 4 | Fall 2021 Midterm 2.3
SPFirst Ch. 7 & 8 | Homework 6.1, 6.3 & 7.2

Problem 5. Discrete-Time Filter Analysis. *12 points.*

Consider the following causal finite impulse response (FIR) linear time-invariant (LTI) filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] - x[n-2]$$

for $n \geq 0$.

(a) What are the initial conditions? What are their values? *3 points*.

SPFirst Sec. 8.2 p. 198

Let n=0: $y[0] = x[0] - x[-2]$
Let n=1: $y[1] = x[1] - x[-1]$
Let n=2: $y[2] = x[2] - x[0]$ etc.
Initial conditions are $x[-1]$ and $x[-2]$ and must be zero for linearity and time-invariant properties to hold.
Note that $x[0]$ is the first input value and not an initial condition, and similarly, $y[0]$ is the first output value and not an initial condition.

(b) Derive the system transfer function $H(z)$ in the z-domain and the region of convergence. *3 points*

Z-transform both sides of the difference equation, knowing that all initial conditions are zero: $Y(z) = X(z) - z^{-2} X(z)$ which means that $H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2}$ for $z \neq 0$.

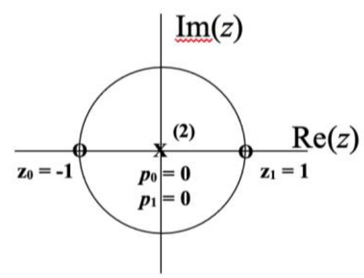
(c) Give a formula for the discrete-time frequency response of the FIR filter. *3 points*.

We can convert the transfer function $H(z)$ into the discrete-time frequency domain by substituting $z = \exp(j\,\omega)$ because the region of convergence includes the unit circle:

$$H_{freq}(\omega) = H(z)]_{z=e^{j\omega}} = 1 - e^{-2j\omega}$$

(d) What is the frequency selectivity: lowpass, highpass, bandpass, bandstop, allpass, notch? *3 points*.

$$H(z) = 1 - z^{-2} = \frac{z^2-1}{z^2} = \frac{(z-1)(z+1)}{z^2}.$$



Zeros at $z = 1$ and $z = -1$. Two poles at $z = 0$.

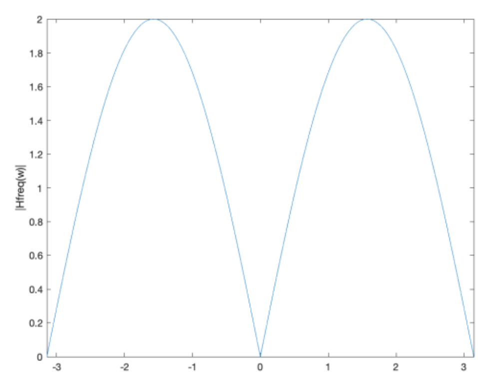
$$H_{freq}(\omega) = \frac{(e^{j\omega} - 1)(e^{j\omega} + 1)}{(e^{j\omega})(e^{j\omega})}$$

Taking the absolute value of both sides gives the magnitude response:

$$|H_{freq}(\omega)| = \left|\frac{(e^{j\omega} - 1)(e^{j\omega} + 1)}{(e^{j\omega})(e^{j\omega})}\right| = \frac{|e^{j\omega} - 1|\,|e^{j\omega} + 1|}{|e^{j\omega}|\,|e^{j\omega}|} = \underbrace{|e^{j\omega} - 1|}_{\substack{zeros\ out \\ \omega=0}}\ \underbrace{|e^{j\omega} + 1|}_{\substack{zeros\ out \\ \omega=\pi}}$$

Zeros at $z = 1$ and $z = -1$ eliminate frequencies at $\omega = 0$ and $\omega = \pi$, respectively. BANDPASS.

```
w = -pi : (2*pi)/1000 : pi;
H = 1 - exp(-j*2*w);
plot(w, abs(H));
xlabel('w');
ylabel('|Hfreq(w)|');
xlim([-pi pi]);
```



Problem 6. Continuous-Time Signal Acrobatics. *12 points.*

Mini-Project #2

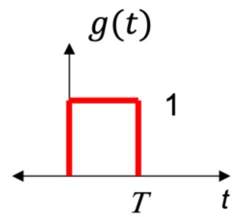
Matched filtering detects a pulse shape in a signal by correlating the signal with the known pulse shape. Applications include communication, radar, sonar, and ultrasound systems.

The matched filter gets its name from its impulse response $h(t)$ being matched to the pulse shape $g(t)$ according to the following formula:

$$h(t) = C\ g^*(T - t)$$

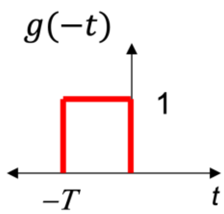
We form $h(t)$ by flipping $g(t)$ in time t, delaying by constant delay T, conjugating the amplitude, and scaling by non-zero constant C. T is often chosen to make the impulse response causal.

For the pulse shape $g(t)$ shown on the right,



(a) Plot $g(-t)$. *4 points.*

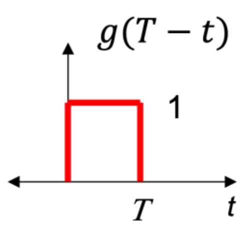
Flip $g(t)$ in time.



(b) Plot $g(T - t)$. This should be a causal signal. *4 points.*

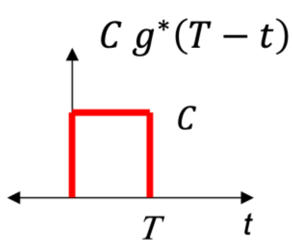
Lecture Slide 2-3

Delay $g(-t)$ by constant delay T.



(c) Plot $C\ g^*(T - t)$. *4 points.*

Conjugating the amplitude of $g^*(T - t)$, which does not have any effect because the pulse shape is real-valued, and then scaling by non-zero constant C.



Fall 2018 Final Exam Problem 7 | SPFirst Ch. 8 IIR Filters

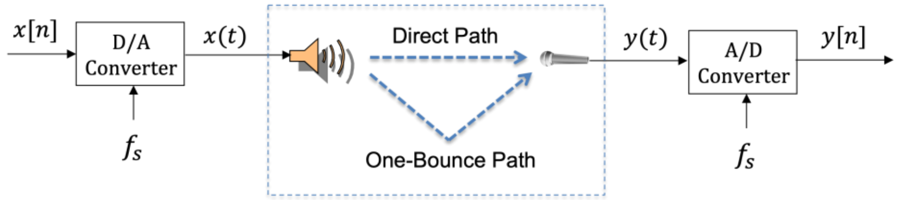
SPFirst Sec. 9-9 Multipath Distortion & Sec. 12-1.5 Compensation

Problem 7. Discrete-Time Equalization. *12 points.*

When sound waves propagate through air, or when electromagnetic waves propagate through air, the waves are absorbed, reflected and scattered by objects in the environment.

In the transmission of sound waves over the air in a room from an audio speaker to a microphone, we will model the direct path from the speaker to the microphone as having zero delay, and a one-bounce path from the speaker to an object and then to the microphone having delay t_1.

This single reflection is a type of echo.



We model the signal $y(t)$ at the output of the microphone as

$$y(t) = x(t) - \alpha\, x(t - t_1)$$

where α is a real-valued constant and $t_1 > 0$.

We model that system that connects $x(t)$ and $y(t)$ as linear and time-invariant (LTI).

By adding a digital-to-analog (D/A) converter on the input the audio speaker and an analog-to-digital converter (A/D) on the output of the microphone, we convert the problem to discrete time:

$$y[n] = x[n] - \alpha\, x[n-1]$$

We're assuming $t_1 = T_s$, α is real-valued, and delay through the D/A and A/D converters is zero.

(a) Derive a formula for the impulse response $h[n]$. *3 points.*

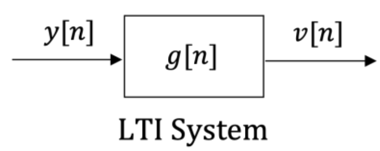
In-Lecture Assignment 6

Impulse response means the system response (output) to an impulse.
Let the input signal be $x[n] = \delta[n]$, then the output signal is $h[n] = \delta[n] - \alpha\,\delta[n-1]$

(b) Find transfer function in the *z*-domain $H(z)$. *3 points.*

Take the z-transform of impulse response: $H(z) = Z\{\, h[n]\,\} = 1 - \alpha\, z^{-1}$ for $z \neq 0$

(c) We add a discrete-time LTI filter at the microphone output to remove as much echo as possible. Design the discrete-time filter by giving its transfer function $G(z)$ in the *z*-domain. The filter $G(z)$ must be bounded-input bounded-output (BIBO) stable. *6 points.*



- Case I. $|\alpha| < 1$. **$H(z)\,G(z) = 1$ so**

$$G(z) = \frac{1}{H(z)} = \frac{1}{1 - \alpha\, z^{-1}}$$

$G(z)$ is BIBO stable because its pole at $z = \alpha$ is inside unit circle.

- Case II. $|\alpha| = 1$. **We cannot use $G(z) = \frac{1}{1-\alpha\, z^{-1}}$. It's not BIBO stable because the pole at $z = \alpha$ is on the unit circle. We move the pole just inside the unit circle to create notch arrangement between the pole at $z = 0.99\alpha$ and zero at $z = \alpha$.**

$$G(z) = \frac{1}{1 - 0.99\,\alpha\, z^{-1}}$$

- Case III. $|\alpha| > 1$. **We cannot use $G(z) = \frac{1}{1-\alpha\, z^{-1}}$. It's not BIBO stable because the pole at $z = \alpha$ is outside the unit circle. If we flip the pole location, i.e. $z = 1/\alpha$, it will be in an allpass configuration with the zero at $z = \alpha$. (If α were complex, then we'd flip the radius and keep the phase as is.)**

$$G(z) = \frac{1}{1 - (1/\alpha)\, z^{-1}}$$

Handout I Allpass Filters

Lectures 5, 6 and 12

Homework 8.3 & 9.1

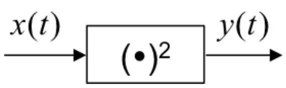
Problem 8. Continuous-Time Frequency-Domain Analysis. *12 points.*

For each problem below, determine the frequency (or frequencies) present in $x(t)$ and $y(t)$ as well as the single sampling rate you would use for the entire system to prevent aliasing.

Please note that $T_c = \frac{1}{f_c}$ and $T_0 = \frac{1}{f_0}$ in the following. *Each part is worth 4* points.

Fall 2018 Final Exam Prob 8(c)

(a) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



Squaring Block Nonlinearity

Fall 2021 Midterm 1.2(a)

Lecture Slide 1-9

Homework 2.4(b)

$x(t)$ has frequencies $-f_c$ and $+f_c$. $y(t) = x^2(t) = ½ + ½ \cos(2\pi (2 f_c) t)$.
Or one could determine $y(t) = x(t)\, x(t)$ by using the Fourier transform

$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * X(j\omega)$ where $X(j\omega) = \pi\, \delta(\omega + \omega_c) + \pi\, \delta(\omega - \omega_c)$

which gives $Y(j\omega) = \frac{\pi}{2} \delta(\omega + 2\omega_c) + \pi\, \delta(\omega) + \frac{\pi}{2} \delta(\omega - 2\omega_c)$ and then

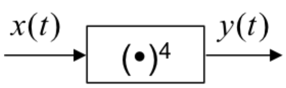
computing the inverse Fourier transform to get $y(t) = ½ + ½ \cos(2\pi (2 f_c) t)$.

$y(t)$ has frequencies $-2f_c$, 0, and $+2f_c$. Here, $f_{max} = 2f_c$.

Sampling Theorem: $f_s > 2 f_{max}$.

Note: Because the component at $2f_c$ in $y(t)$ is a cosine, one could use $f_s \geq 2 f_{max}$.

(c) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



Fourier-Order Nonlinearity

Homework 2.4(c)

$x(t)$ has frequencies $-f_c$ and $+f_c$. Cascade of two squaring blocks:

$$y(t) = (x^2(t))^2 = \left(\frac{1}{2} + \frac{1}{2}\cos(2\pi(2f_c)t)\right)^2$$

$$y(t) = \frac{1}{4} + \frac{1}{2}\cos(2\pi(2f_c)t) + \frac{1}{4}\cos^2(2\pi(2f_c)t)$$

$$y(t) = \frac{3}{8} + \frac{1}{2}\cos(2\pi(2f_c)t) + \frac{1}{8}\cos(2\pi(4f_c)t)$$

First squarer gives $-2f_c$, 0, and $+2f_c$ from part (a).
Second squarer gives $-4f_c$, $-2f_c$, 0, $+2f_c$, $+4f_c$.

Here, $f_{max} = 4f_c$. Choose $f_s > 2 f_{max}$.

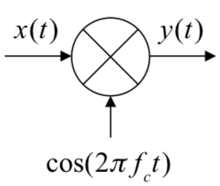
(d) Let $x(t) = \text{sinc}\left(\frac{t}{T_0}\right) = \frac{\sin\left(\pi \frac{t}{T_0}\right)}{\pi \frac{t}{T_0}}$

be a continuous-time signal for $-\infty < t < \infty$

whose continuous-time Fourier transform is

$$X(f) = T_0 \,\text{rect}\left(\frac{f}{f_0}\right)$$

Here, $f_c > f_0$



Fall 2021 Midterm 1.3

Homework 2.2

In-Lecture Assignment #2

Sinusoidal amplitude modulation:

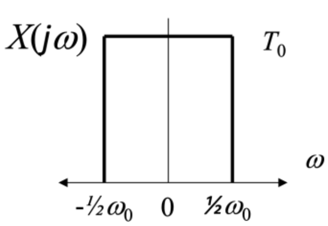
$$y(t) = \text{sinc}\left(\frac{t}{T_0}\right)\cos(2\pi f_c t) = x(t)\cos(2\pi f_c t)$$

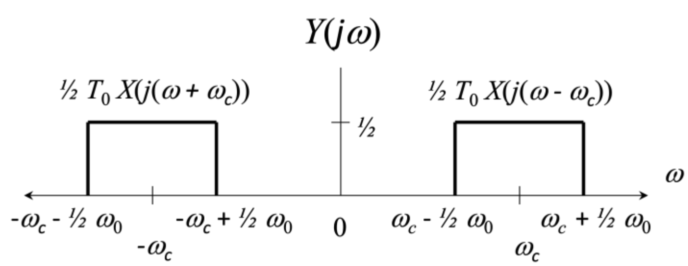
Modulation property for the Fourier transform:

$$Y(j\omega) = \frac{1}{2} X(j(\omega + \omega_0)) + \frac{1}{2} X(j(\omega - \omega_0))$$

Frequency content of $X(j\omega)$ shifts left & right by ω_0.
From the Fourier transform table on *SPFirst* p. 338,

$$X(j\omega) = T_0 \,\text{rect}\left(\frac{\omega}{\omega_0}\right)$$





$y(t)$ has frequencies $[-f_c - \frac{1}{2} f_0, -f_c + \frac{1}{2} f_0]$ and $[f_c - \frac{1}{2} f_0, f_c + \frac{1}{2} f_0]$.

Here, $f_{max} = f_c + \frac{1}{2} f_0$ and choose $f_s > 2\, f_{max}$ to satisfy Sampling Theorem